(B) All rational numbers are real numbers.

(C) All natural numbers are rational numbers.

45. Give an example of a rational number that is not an integer.

46. Give an example of a real number that is not a rational number.

47. Given the sets of numbers N (natural numbers), Z (integers), Q (rational numbers), and R (real numbers), indicate to which set(s) each of the following numbers belongs:

(A) -3 (B) 3.14 (C) π (D) $\frac{2}{3}$

48. Given the sets of numbers N, Z, Q, and R (see Problem 47), indicate to which set(s) each of the following numbers belongs:

(A) 8 (B) $\sqrt{2}$ (C) -1.414 (D) $\frac{-5}{2}$

In Problems 49 and 50, use a calculator to express each number as a decimal fraction to the capacity of your calculator (refer to the user's manual for your calculator). Observe the repeating decimal representation of the rational numbers and the apparent nonrepeating decimal representation of the irrational numbers.

49. (A) $\frac{8}{9}$ (B) $\frac{3}{11}$ (C) $\sqrt{5}$ (D) $\frac{11}{8}$

50. (A) $\frac{13}{6}$ (B) $\sqrt{21}$ (C) $\frac{7}{16}$ (D) $\frac{29}{111}$

51. Indicate true (T) or false (F), and for each false statement find real number replacements for a and b that will provide a counterexample. For all real numbers a and b:

(A) $a + b = b + a$ (B) $a - b = b - a$

(C) $ab = ba$ (D) $a \div b = b \div a$

52. Indicate true (T) or false (F), and for each false statement find real number replacements for a, b, and c that will provide a counterexample. For all real numbers a, b, and c:

(A) $(a + b) + c = a + (b + c)$

(B) $(a - b) - c = a - (b - c)$

(C) $a(bc) = (ab)c$

(D) $(a \div b) \div c = a \div (b \div c)$

C

53. If $A = \{1, 2, 3, 4\}$ and $B = \{2, 4, 6\}$, find

(A) $\{x \mid x \in A \text{ or } x \in B\}$

(B) $\{x \mid x \in A \text{ and } x \in B\}$

54. If $F = \{-2, 0, 2\}$ and $G = \{-1, 0, 1, 2\}$, find

(A) $\{x \mid x \in F \text{ or } x \in G\}$

(B) $\{x \mid x \in F \text{ and } x \in G\}$

55. If $c = 0.151\,515\ldots$, then $100c = 15.1515\ldots$ and

$$100c - c = 15.1515\ldots - 0.151\,515\ldots$$
$$99c = 15$$
$$c = \frac{15}{99} = \frac{5}{33}$$

Proceeding similarly, convert the repeating decimal $0.090909\ldots$ into a fraction. (All repeating decimals are rational numbers, and all rational numbers have repeating decimal representations.)

56. Repeat Problem 55 for $0.181\,818\ldots$.

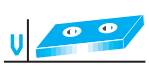
57. To see how the distributive property is behind the mechanics of long multiplication, compute each of the following and compare:

Long Multiplication	Use of the Distributive Property
23	$23 \cdot 12$
$\times\ 12$	$= 23(2 + 10)$
	$= 23 \cdot 2 + 23 \cdot 10 =$

58. For a and b real numbers, justify each step using a property in this section.

Statement		Reason
1.	$(a + b) + (-a) = (-a) + (a + b)$	**1.**
2.	$= [(-a) + a] + b$	**2.**
3.	$= 0 + b$	**3.**
4.	$= b$	**4.**

Section A-2 Polynomials: Basic Operations



- Natural Number Exponents
- Polynomials
- Combining Like Terms
- Addition and Subtraction
- Multiplication
- Combined Operations
- Application

In this section we review the basic operations on *polynomials*, a mathematical form encountered frequently throughout mathematics. We start the discussion with a brief review of natural number exponents. Integer and rational exponents and their properties will be discussed in detail in subsequent sections.

Natural Number Exponents

The definition of a **natural number exponent** is given below.

DEFINITION 1

NATURAL NUMBER EXPONENT

For n a natural number and a any real number,

$$a^n = \underbrace{a \cdot a \cdot \cdots \cdot a}_{n \text{ factors of } a} \qquad 2^4 = \underbrace{2 \cdot 2 \cdot 2 \cdot 2}_{4 \text{ factors of } 2}$$

Also, the **first property of exponents** is stated as follows:

THEOREM 1

FIRST PROPERTY OF EXPONENTS

For any natural numbers m and n, and any real number a,

$$a^m a^n = a^{m+n} \qquad (3x^5)(2x^7) = (3 \cdot 2)x^{5+7} = 6x^{12}$$

Polynomials

Algebraic expressions are formed by using constants and variables and the algebraic operations of addition, subtraction, multiplication, division, raising to powers, and taking roots. Some examples are

$$\sqrt[3]{x^3 + 5} \qquad 5x^4 + 2x^2 - 7$$

$$x + y - 7 \qquad (2x - y)^2$$

$$\frac{x - 5}{x^2 + 2x - 5} \qquad 1 + \cfrac{1}{1 + \cfrac{1}{x}}$$

An algebraic expression involving only the operations of addition, subtraction, multiplication, and raising to natural number powers on variables and constants is called a **polynomial.** Some examples are

$$2x - 3 \qquad 4x^2 - 3x + 7$$

$$x - 2y \qquad 5x^3 - 2x^2 - 7x + 9$$

$$5 \qquad x^2 - 3xy + 4y^2$$

$$0 \qquad x^3 - 3x^2y + xy^2 + 2y^7$$

In a polynomial, a variable cannot appear in a denominator, as an exponent, or within a radical. Accordingly, a **polynomial in one variable** x is constructed by adding or subtracting constants and terms of the form ax^n, where a is a real number and n is a natural number. A **polynomial in two variables** x and y is constructed by adding and subtracting constants and terms of the form ax^my^n, where a is a real number and m and n are natural numbers. Polynomials in three or more variables are defined in a similar manner.

Polynomial forms can be classified according to their *degree*. If a term in a polynomial has only one variable as a factor, then the **degree of that term** is the power of the variable. If two or more variables are present in a term as factors, then the **degree of the term** is the sum of the powers of the variables. The **degree of a polynomial** is the degree of the nonzero term with the highest degree in the polynomial. Any nonzero constant is defined to be a **polynomial of degree 0.** The number 0 is also a polynomial but is not assigned a degree.

EXAMPLE 1 **Polynomials and Nonpolynomials**

(A) Polynomials in one variable:

$$x^2 - 3x + 2 \qquad 6x^3 - \sqrt{2}x - \tfrac{1}{3}$$

(B) Polynomials in several variables:

$$3x^2 - 2xy + y^2 \qquad 4x^3y^2 - \sqrt{3}xy^2z^5$$

(C) Nonpolynomials:

$$\sqrt{2x} - \frac{3}{x} + 5 \qquad \frac{x^2 - 3x + 2}{x - 3} \qquad \sqrt{x^2 - 3x + 1}$$

(D) The degree of the first term in $6x^3 - \sqrt{2}x - \frac{1}{3}$ is 3, the degree of the second term is 1, the degree of the third term is 0, and the degree of the whole polynomial is 3.

(E) The degree of the first term in $4x^3y^2 - \sqrt{3}xy^2$ is 5, the degree of the second term is 3, and the degree of the whole polynomial is 5.

MATCHED PROBLEM 1

(A) Which of the following are polynomials?

$$3x^2 - 2x + 1 \qquad \sqrt{x - 3} \qquad x^2 - 2xy + y^2 \qquad \frac{x - 1}{x^2 + 2}$$

(B) Given the polynomial $3x^5 - 6x^3 + 5$, what is the degree of the first term? The second term? The whole polynomial?

(C) Given the polynomial $6x^4y^2 - 3xy^3$, what is the degree of the first term? The second term? The whole polynomial?

In addition to classifying polynomials by degree, we also call a single-term polynomial a **monomial,** a two-term polynomial a **binomial,** and a three-term polynomial a **trinomial.**

$\frac{5}{2}x^2y^3$	Monomial
$x^3 + 4.7$	Binomial
$x^4 - \sqrt{2}x^2 + 9$	Trinomial

Combining Like Terms

We start with a word about *coefficients.* A constant in a term of a polynomial, including the sign that precedes it, is called the **numerical coefficient,** or simply, the **coefficient,** of the term. If a constant doesn't appear, or only a + sign appears, the coefficient is understood to be 1. If only a − sign appears, the coefficient is understood to be −1. Thus, given the polynomial

$$2x^4 - 4x^3 + x^2 - x + 5 \qquad 2x^4 + (-4)x^3 + 1x^2 + (-1)x + 5$$

the coefficient of the first term is 2, the coefficient of the second term is −4, the coefficient of the third term is 1, the coefficient of the fourth term is −1, and the coefficient of the last term is 5.

At this point, it is useful to state two additional distributive properties of real numbers that follow from the distributive properties stated in Section A-1.

ADDITIONAL DISTRIBUTIVE PROPERTIES

1. $a(b - c) = (b - c)a = ab - ac$
2. $a(b + c + \cdots + f) = ab + ac + \cdots + af$

Two terms in a polynomial are called **like terms** if they have exactly the same variable factors to the same powers. The numerical coefficients may or may not be the same. Since constant terms involve no variables, all constant terms are like terms. If a polynomial contains two or more like terms, these terms can be combined into a single term by making use of distributive properties. Consider the following example:

$$\begin{aligned} 5x^3y - 2xy - x^3y - 2x^3y &= 5x^3y - x^3y - 2x^3y - 2xy \\ &= (5x^3y - x^3y - 2x^3y) - 2xy \\ &= (5 - 1 - 2)x^3y - 2xy \\ &= 2x^3y - 2xy \end{aligned}$$

It should be clear that free use has been made of the real number properties discussed earlier. The steps done in the dashed box are usually done mentally, and the process is quickly mechanized as follows:

Like terms in a polynomial are combined by adding their numerical coefficients.

EXAMPLE 2

Simplifying Polynomials

Remove parentheses and combine like terms.

(A) $2(3x^2 - 2x + 5) + (x^2 + 3x - 7)$

$$= 2(3x^2 - 2x + 5) + 1(x^2 + 3x - 7)$$

Think

$$= 6x^2 - 4x + 10 + x^2 + 3x - 7$$

$$= 7x^2 - x + 3$$

(B) $(x^3 - 2x - 6) - (2x^3 - x^2 + 2x - 3)$

$$= 1(x^3 - 2x - 6) + (-1)(2x^3 - x^2 + 2x - 3)$$

Think

Be careful with the sign here.

$$= x^3 - 2x - 6 - 2x^3 + x^2 - 2x + 3$$

$$= -x^3 + x^2 - 4x - 3$$

(C) $[3x^2 - (2x + 1)] - (x^2 - 1) = [3x^2 - 2x - 1] - (x^2 - 1)$

Remove inner parentheses first.

$$= 3x^2 - 2x - 1 - x^2 + 1$$

$$= 2x^2 - 2x$$

MATCHED PROBLEM 2

Remove parentheses and combine like terms.

(A) $3(u^2 - 2v^2) + (u^2 + 5v^2)$

(B) $(m^3 - 3m^2 + m - 1) - (2m^3 - m + 3)$

(C) $(x^3 - 2) - [2x^3 - (3x + 4)]$

Addition and Subtraction

Addition and subtraction of polynomials can be thought of in terms of removing parentheses and combining like terms, as illustrated in Example 2. Horizontal and vertical arrangements are illustrated in the next two examples. You should be able to work either way, letting the situation dictate the choice.

EXAMPLE 3

Adding Polynomials

Add.

$$x^4 - 3x^3 + x^2, \quad -x^3 - 2x^2 + 3x, \quad \text{and} \quad 3x^2 - 4x - 5$$

Solution

Add horizontally.

$$\begin{aligned}(x^4 - 3x^3 + x^2) + (-x^3 - 2x^2 + 3x) + (3x^2 - 4x - 5)\\ = x^4 - 3x^3 + x^2 - x^3 - 2x^2 + 3x + 3x^2 - 4x - 5\\ = x^4 - 4x^3 + 2x^2 - x - 5\end{aligned}$$

Or vertically, by lining up like terms and adding their coefficients.

$$\begin{array}{r} x^4 - 3x^3 + x^2 \\ -\ x^3 - 2x^2 + 3x \\ \hline 3x^2 - 4x - 5 \\ \hline x^4 - 4x^3 + 2x^2 - x - 5 \end{array}$$

MATCHED PROBLEM 3

Add horizontally and vertically.

$3x^4 - 2x^3 - 4x^2$, $x^3 - 2x^2 - 5x$, and $x^2 + 7x - 2$

EXAMPLE 4

Subtracting Polynomials

Subtract.

$4x^2 - 3x + 5$ from $x^2 - 8$

Solution

$$\begin{aligned}(x^2 - 8) - (4x^2 - 3x + 5) &\quad \text{or} \quad x^2 - 8\\ = x^2 - 8 - 4x^2 + 3x - 5 &\quad \underline{-4x^2 + 3x - 5} \quad \leftarrow \text{Change signs and add.}\\ = -3x^2 + 3x - 13 &\quad -3x^2 + 3x - 13\end{aligned}$$

MATCHED PROBLEM 4

Subtract.

$2x^2 - 5x + 4$ from $5x^2 - 6$

CAUTION

When you use a horizontal arrangement to subtract a polynomial with more than one term, you must enclose the polynomial in parentheses. Thus, to subtract $2x + 5$ from $4x - 11$, you must write

$4x - 11 - (2x + 5)$ and not $4x - 11 - 2x + 5$

Multiplication

Multiplication of algebraic expressions involves the extensive use of distributive properties for real numbers, as well as other real number properties.

EXAMPLE 5 **Multiplying Polynomials**

Multiply.

$$(2x - 3)(3x^2 - 2x + 3)$$

Solution

$$\begin{aligned}(2x - 3)(3x^2 - 2x + 3) &= 2x(3x^2 - 2x + 3) - 3(3x^2 - 2x + 3)\\ &= 6x^3 - 4x^2 + 6x - 9x^2 + 6x - 9\\ &= 6x^3 - 13x^2 + 12x - 9\end{aligned}$$

or, using a vertical arrangement,

$$\begin{array}{r} 3x^2 - 2x + 3 \\ \underline{2x \quad - 3} \\ 6x^3 - 4x^2 + 6x \\ \underline{\quad - 9x^2 + 6x - 9} \\ 6x^3 - 13x^2 + 12x - 9 \end{array}$$

MATCHED PROBLEM 5

Multiply.

$$(2x - 3)(2x^2 + 3x - 2)$$

Thus, to multiply two polynomials, multiply each term of one by each term of the other, and combine like terms.

Products of certain binomial factors occur so frequently that it is useful to develop procedures that will enable us to write down their products by inspection. To find the product $(2x - 1)(3x + 2)$, we will use the popular **FOIL method.** We multiply each term of one factor by each term of the other factor as follows:

	F First product ↓	O Outer product ↓	I Inner product ↓	L Last product ↓
$(2x - 1)(3x + 2) =$	$6x^2$	$+ 4x$	$- 3x$	$- 2$

The inner and outer products are like terms and hence combine into one term. Thus,

$$(2x - 1)(3x + 2) = 6x^2 + x - 2$$

To speed up the process, we combine the inner and outer product mentally.

Products of certain binomial factors occur so frequently that it is useful to remember formulas for their products. The following formulas are easily verified by multiplying the factors on the left using the FOIL method:

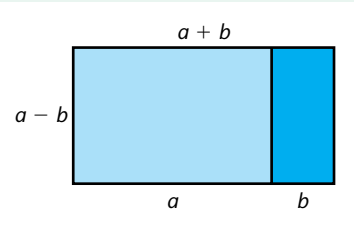
SPECIAL PRODUCTS

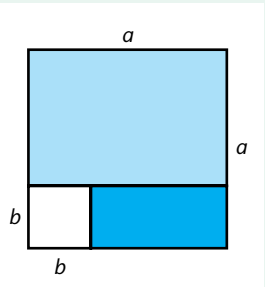
1. $(a - b)(a + b) = a^2 - b^2$
2. $(a + b)^2 = a^2 + 2ab + b^2$
3. $(a - b)^2 = a^2 - 2ab + b^2$

Explore/Discuss 1

(A) Explain the relationship between special product formula 1 and the areas of the rectangles in the figures.

$$(a - b)(a + b) \quad = \quad a^2 - b^2$$





(B) Construct similar figures to provide geometric interpretations for special product formulas 2 and 3.

EXAMPLE 6 Multiplying Binomials

Multiply.

(A) $(2x - 3y)(5x + 2y)$ $= 10x^2 + 4xy - 15xy - 6y^2$

$= 10x^2 - 11xy - 6y^2$

(B) $(3a - 2b)(3a + 2b)$ $= (3a)^2 - (2b)^2$ $= 9a^2 - 4b^2$

(C) $(5x - 3)^2$ $= (5x)^2 - 2(5x)(3) + 3^2$ $= 25x^2 - 30x + 9$

(D) $(m + 2n)^2 = m^2 + 4mn + 4n^2$

MATCHED PROBLEM 6

Multiply.

(A) $(4u - 3v)(2u + v)$ (B) $(2xy + 3)(2xy - 3)$

(C) $(m + 4n)(m - 4n)$ (D) $(2u - 3v)^2$ (E) $(6x + y)^2$

CAUTION

Remember to include the sum of the inner and outer terms when using the FOIL method to square a binomial. That is,

$$(x + 3)^2 \neq x^2 + 9 \qquad (x + 3)^2 = x^2 + 6x + 9$$

Combined Operations

We now consider several examples that use all the operations just discussed. Before considering these examples, it is useful to summarize order-of-operation conventions pertaining to exponents, multiplication and division, and addition and subtraction.

ORDER OF OPERATIONS

1. Simplify inside the innermost grouping first, then the next innermost, and so on.

$$\begin{aligned} 2[3 - (x - 4)] &= 2[3 - x + 4] \\ &= 2(7 - x) = 14 - 2x \end{aligned}$$

2. Unless grouping symbols indicate otherwise, apply exponents before multiplication or division is performed.

$$2(x - 2)^2 = 2(x^2 - 4x + 4) = 2x^2 - 8x + 8$$

3. Unless grouping symbols indicate otherwise, perform multiplication and division before addition and subtraction. In either case, proceed from left to right.

$$5 - 2(x - 3) = 5 - 2x + 6 = 11 - 2x$$

EXAMPLE 7 Combined Operations

Perform the indicated operations and simplify.

(A) $3x - \{5 - 3[x - x(3 - x)]\}$

$$\begin{aligned} &= 3x - \{5 - 3[x - 3x + x^2]\} \\ &= 3x - \{5 - 3[-2x + x^2]\} \\ &= 3x - \{5 + 6x - 3x^2\} \\ &= 3x - 5 - 6x + 3x^2 \\ &= 3x^2 - 3x - 5 \end{aligned}$$

(B) $(x - 2y)(2x + 3y) - (2x + y)^2$

$$\begin{aligned} &= 2x^2 + 3xy - 4xy - 6y^2 \\ &\quad - (4x^2 + 4xy + y^2) \\ &= 2x^2 - xy - 6y^2 - 4x^2 - 4xy - y^2 \\ &= -2x^2 - 5xy - 7y^2 \end{aligned}$$

$$\begin{aligned}(C)\ (2m + 3n)^3 &= (2m + 3n)(2m + 3n)^2\\ &= (2m + 3n)(4m^2 + 12mn + 9n^2)\\ &= 8m^3 + 24m^2n + 18mn^2 + 12m^2n + 36mn^2 + 27n^3\\ &= 8m^3 + 36m^2n + 54mn^2 + 27n^3\end{aligned}$$

MATCHED PROBLEM 7

Perform the indicated operations and simplify.

(A) $2t - \{7 - 2[t - t(4 + t)]\}$ (B) $(u - 3v)^2 - (2u - v)(2u + v)$

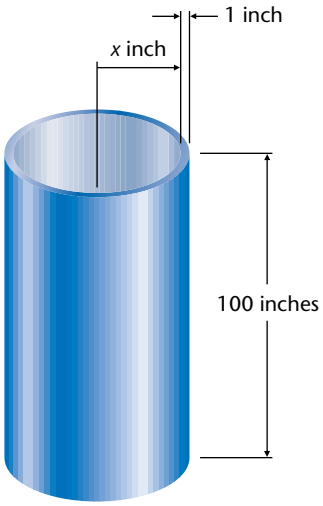
(C) $(4x - y)^3$

Application

EXAMPLE 8 C|∫

Volume of a Cylindrical Shell

A plastic water pipe with a hollow center is 100 inches long, 1 inch thick, and has an inner radius of x inches (see the figure). Write an algebraic expression in terms of x that represents the volume of the plastic used to construct the pipe. Simplify the expression. [*Recall:* The volume V of a right circular cylinder of radius r and height h is given by $V = \pi r^2 h$.]



Solution

A right circular cylinder with a hollow center is called a **cylindrical shell.** The volume of the shell is equal to the volume of the cylinder minus the volume of the hole. Since the radius of the hole is x inches and the pipe is 1 inch thick, the radius of the cylinder is $x + 1$ inches. Thus, we have

$$\begin{pmatrix}\text{Volume of}\\ \text{shell}\end{pmatrix} = \begin{pmatrix}\text{Volume of}\\ \text{cylinder}\end{pmatrix} - \begin{pmatrix}\text{Volume of}\\ \text{hole}\end{pmatrix}$$

$$\begin{aligned}\text{Volume} &= \pi(x + 1)^2\, 100 - \pi x^2 100\\ &= 100\pi(x^2 + 2x + 1) - 100\pi x^2\\ &= 100\pi x^2 + 200\pi x + 100\pi - 100\pi x^2\\ &= 200\pi x + 100\pi\end{aligned}$$

MATCHED PROBLEM 8

A plastic water pipe is 200 inches long, 2 inches thick, and has an outer radius of x inches. Write an algebraic expression in terms of x that represents the volume of the plastic used to construct the pipe. Simplify the expression.

Answers to Matched Problems

1. (A) $3x^2 - 2x + 1$, $x^2 - 2xy + y^2$ (B) 5, 3, 5 (C) 6, 4, 6
2. (A) $4u^2 - v^2$ (B) $-m^3 - 3m^2 + 2m - 4$ (C) $-x^3 + 3x + 2$
3. $3x^4 - x^3 - 5x^2 + 2x - 2$ **4.** $3x^2 + 5x - 10$ **5.** $4x^3 - 13x + 6$
6. (A) $8u^2 - 2uv - 3v^2$ (B) $4x^2y^2 - 9$ (C) $m^2 - 16n^2$ (D) $4u^2 - 12uv + 9v^2$ (E) $36x^2 + 12xy + y^2$
7. (A) $-2t^2 - 4t - 7$ (B) $-3u^2 - 6uv + 10v^2$ (C) $64x^3 - 48x^2y + 12xy^2 - y^3$
8. Volume $= 200\pi x^2 - 200\pi(x - 2)^2 = 800\pi x - 800\pi$

EXERCISE A-2

A

Problems 1–8 refer to the following polynomials:
(a) $2x^3 - 3x^2 + x + 5$ *(b)* $2x^2 + x - 1$ *(c)* $3x - 2$

1. What is the degree of (a)?
2. What is the degree of (b)?
3. Add (a) and (b).
4. Add (b) and (c).
5. Subtract (b) from (a).
6. Subtract (c) from (b).
7. Multiply (a) and (c).
8. Multiply (b) and (c).

In Problems 9–28, perform the indicated operations and simplify.

9. $2(x - 1) + 3(2x - 3) - (4x - 5)$
10. $2(u - 1) - (3u + 2) - 2(2u - 3)$
11. $2y - 3y[4 - 2(y - 1)]$
12. $4a - 2a[5 - 3(a + 2)]$
13. $(m - n)(m + n)$
14. $(a + b)(a - b)$
15. $(4t - 3)(t - 2)$
16. $(3x - 5)(2x + 1)$
17. $(3x + 2y)(x - 3y)$
18. $(2x - 3y)(x + 2y)$
19. $(2m - 7)(2m + 7)$
20. $(3y + 2)(3y - 2)$
21. $(6x - 4y)(5x + 3y)$
22. $(3m + 7n)(2m - 5n)$
23. $(3x - 2y)(3x + 2y)$
24. $(4m + 3n)(4m - 3n)$
25. $(4x - y)^2$
26. $(3u + 4v)^2$
27. $(a + b)(a^2 - ab + b^2)$
28. $(a - b)(a^2 + ab + b^2)$

B

In Problems 29–42, perform the indicated operations and simplify.

29. $2x - 3\{x + 2[x - (x + 5)] + 1\}$
30. $m - \{m - [m - (m - 1)]\}$
31. $2\{3[a - 4(1 - a)] - (5 - a)\}$
32. $5b - 3\{-[2 - 4(2b - 1)] + 2(2 - 3b)\}$
33. $(2x^2 + x - 2)(x^2 - 3x + 5)$
34. $(x^2 - 2xy + y^2)(x^2 + 2xy + y^2)$
35. $(h^2 + hk + k^2)(h^2 - hk + k^2)$
36. $(n^2 + 2n + 1)(n^2 - 4n - 3)$
37. $(2x - 1)^2 - (3x + 2)(3x - 2)$
38. $(3a - b)(3a + b) - (2a - 3b)^2$
39. $(m - 3n)(m + 8n) + (m + 6n)(m + 4n)$
40. $(y - 2)(y + 1) + (y - 3)(y + 4)$
41. $(2m - n)^3$
42. $(3a + 2b)^3$

C | ∫

Problems 43–50 are calculus-related. Perform the indicated operations and simplify.

43. $5(x + h) - 4 - (5x - 4)$
44. $6(x + h) + 2 - (6x + 2)$
45. $3(x + h)^2 + 2(x + h) - (3x^2 + 2x)$
46. $4(x + h)^2 - 5(x + h) - (4x^2 - 5x)$
47. $-2(x + h)^2 - 3(x + h) + 7 - (-2x^2 - 3x + 7)$
48. $-(x + h)^2 + 4(x + h) - 9 - (-x^2 + 4x - 9)$
49. $(x + h)^3 - x^3$
50. $2(x + h)^2 + 3(x + h) - (2x^2 + 3x)$
51. Subtract the sum of the first two polynomials from the sum of the last two: $3m^2 - 2m + 5$, $4m^2 - m$, $3m^2 - 3m - 2$, $m^3 + m^2 + 2$
52. Subtract the sum of the last two polynomials from the sum of the first two: $2x^2 - 4xy + y^2$, $3xy - y^2$, $x^2 - 2xy - y^2$, $-x^2 + 3xy - 2y^2$

C

In Problems 53–56, perform the indicated operations and simplify.

53. $2(x-2)^3-(x-2)^2-3(x-2)-4$

54. $(2x-1)^3-2(2x-1)^2+3(2x-1)+7$

55. $-3x\{x[x-x(2-x)]-(x+2)(x^2-3)\}$

56. $2\{(x-3)(x^2-2x+1)-x[3-x(x-2)]\}$

57. Show by example that, in general, $(a+b)^2 \neq a^2+b^2$. Discuss possible conditions on a and b that would make this a valid equation.

58. Show by example that, in general, $(a-b)^2 \neq a^2-b^2$. Discuss possible conditions on a and b that would make this a valid equation.

59. If you are given two polynomials, one of degree m and the other of degree n, $m > n$, what is the degree of the sum?

60. What is the degree of the product of the two polynomials in Problem 59?

61. How does the answer to Problem 59 change if the two polynomials can have the same degree?

62. How does the answer to Problem 60 change if the two polynomials can have the same degree?

APPLICATIONS

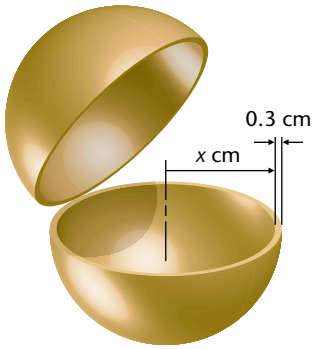
63. Geometry. The width of a rectangle is 5 centimeters less than its length. If x represents the length, write an algebraic expression in terms of x that represents the perimeter of the rectangle. Simplify the expression.

64. Geometry. The length of a rectangle is 8 meters more than its width. If x represents the width of the rectangle, write an algebraic expression in terms of x that represents its area. Change the expression to a form without parentheses.

★ **65. Coin Problem.** A parking meter contains nickels, dimes, and quarters. There are 5 fewer dimes than nickels, and 2 more quarters than dimes. If x represents the number of nickels, write an algebraic expression in terms of x that represents the value of all the coins in the meter in cents. Simplify the expression.

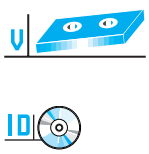
★ **66. Coin Problem.** A vending machine contains dimes and quarters only. There are 4 more dimes than quarters. If x represents the number of quarters, write an algebraic expression in terms of x that represents the value of all the coins in the vending machine in cents. Simplify the expression.

67. Packaging. A spherical plastic container for designer wristwatches has an inner radius of x centimeters (see the figure). If the plastic shell is 0.3 centimeters thick, write an algebraic expression in terms of x that represents the volume of the plastic used to construct the container. Simplify the expression. [*Recall:* The volume V of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.]



68. Packaging. A cubical container for shipping computer components is formed by coating a metal mold with polystyrene. If the metal mold is a cube with sides x centimeters long and the polystyrene coating is 2 centimeters thick, write an algebraic expression in terms of x that represents the volume of the polystyrene used to construct the container. Simplify the expression. [*Recall:* The volume V of a cube with sides of length t is given by $V = t^3$.]

Section A-3 | Polynomials: Factoring



- Factoring—What Does It Mean?
- Common Factors and Factoring by Grouping
- Factoring Second-Degree Polynomials
- More Factoring

Factoring—What Does It Mean?

A **factor of a number** is one of two or more numbers whose product is the given number. Similarly, a **factor of an algebraic expression** is one of two or more algebraic expressions whose product is the given algebraic expression. For example,